Seventh Edition in SI Units

MECHANICS OF MATERIALS

Ferdinand Beer · E. Russel Johnston, Jr. · John DeWolf · David Mazurek

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Mechanics of Materials

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Contents

Preface ix
Guided Tour xiii
List of Symbols xv

1 Introduction—Concept of Stress 3

1.1 Review of The Methods of Statics 4
1.2 Stresses in the Members of a Structure 7
1.3 Stress on an Oblique Plane Under Axial Loading 27
1.4 Stress Under General Loading Conditions; Components of Stress 28
1.5 Design Considerations 31
Review and Summary 44

2 Stress and Strain—Axial Loading 55

2.1 An Introduction to Stress and Strain 57
2.2 Statically Indeterminate Problems 78
2.3 Problems Involving Temperature Changes 82
2.4 Poisson's Ratio 94
2.5 Multiaxial Loading: Generalized Hooke's Law 95
*2.6 Dilatation and Bulk Modulus 97
2.7 Shearing Strain 99
2.8 Deformations Under Axial Loading—Relation Between E, ν, and G 102
*2.9 Stress-Strain Relationships For Fiber-Reinforced Composite Materials 104
2.10 Stress and Strain Distribution Under Axial Loading: Saint-Venant's Principle 115
2.11 Stress Concentrations 117
2.12 Plastic Deformations 119
*2.13 Residual Stresses 123
Review and Summary 133

*Advanced or specialty topics

Contents

Preface ix
Guided Tour xiii
List of Symbols xv

1 Introduction—Concept of Stress 3

1.1 Review of The Methods of Statics 4
1.2 Stresses in the Members of a Structure 7
1.3 Stress on an Oblique Plane Under Axial Loading 27
1.4 Stress Under General Loading Conditions; Components of Stress 28
1.5 Design Considerations 31
Review and Summary 44

2 Stress and Strain—Axial Loading 55

2.1 An Introduction to Stress and Strain 57
2.2 Statically Indeterminate Problems 78
2.3 Problems Involving Temperature Changes 82
2.4 Poisson's Ratio 94
2.5 Multiaxial Loading: Generalized Hooke's Law 95
*2.6 Dilatation and Bulk Modulus 97
2.7 Shearing Strain 99
2.8 Deformations Under Axial Loading—Relation Between E, ν, and G 102
*2.9 Stress-Strain Relationships For Fiber-Reinforced Composite Materials 104
2.10 Stress and Strain Distribution Under Axial Loading: Saint-Venant's Principle 115
2.11 Stress Concentrations 117
2.12 Plastic Deformations 119
*2.13 Residual Stresses 123
Review and Summary 133

*Advanced or specialty topics

3 Torsion 147

3.1 Circular Shafts in Torsion 150
3.2 Angle of Twist in the Elastic Range 167
3.3 Statically Indeterminate Shafts 170
3.4 Design of Transmission Shafts 185
3.5 Stress Concentrations in Circular Shafts 187
*3.6 Plastic Deformations in Circular Shafts 195
*3.7 Circular Shafts Made of an Elastoplastic Material 196
*3.8 Residual Stresses in Circular Shafts 199
*3.9 Torsion of Noncircular Members 209
*3.10 Thin-Walled Hollow Shafts 211
Review and Summary 223

4 Pure Bending 237

4.1 Symmetric Members in Pure Bending 240
4.2 Stresses and Deformations in the Elastic Range 244
4.3 Deformations in a Transverse Cross Section 248
4.4 Members Made of Composite Materials 259
4.5 Stress Concentrations 263
*4.6 Plastic Deformations 273
4.7 Eccentric Axial Loading in a Plane of Symmetry 291
4.8 Unsymmetric Bending Analysis 302
4.9 General Case of Eccentric Axial Loading Analysis 307
*4.10 Curved Members 319
Review and Summary 334

5 Analysis and Design of Beams for Bending 345

5.1 Shear and Bending-Moment Diagrams 348
5.2 Relationships Between Load, Shear, and Bending Moment 360
5.3 Design of Prismatic Beams for Bending 371
*5.4 Singularity Functions Used to Determine Shear and Bending Moment 383
*5.5 Nonprismatic Beams 396
Review and Summary 407

6 Shearing Stresses in Beams and Thin-Walled Members 417

6.1 Horizontal Shearing Stress in Beams 420
*6.2 Distribution of Stresses in a Narrow Rectangular Beam 426
6.3 Longitudinal Shear on a Beam Element of Arbitrary Shape 437
6.4 Shearing Stresses in Thin-Walled Members 439
*6.5 Plastic Deformations 441
*6.6 Unsymmetric Loading of Thin-Walled Members and Shear Center 454

Review and Summary 467

7 Transformations of Stress and Strain 477

7.1 Transformation of Plane Stress 480
7.2 Mohr's Circle for Plane Stress 492
7.3 General State of Stress 503
7.4 Three-Dimensional Analysis of Stress 504
*7.5 Theories of Failure 507
7.6 Stresses in Thin-Walled Pressure Vessels 520
*7.7 Transformation of Plane Strain 529
*7.8 Three-Dimensional Analysis of Strain 534
*7.9 Measurements of Strain; Strain Rosette 538

Review and Summary 546

8 Principal Stresses Under a Given Loading 557

8.1 Principal Stresses in a Beam 559
8.2 Design of Transmission Shafts 562
8.3 Stresses Under Combined Loads 575

Review and Summary 591

9 Deflection of Beams 599

9.1 Deformation Under Transverse Loading 602
9.2 Statically Indeterminate Beams 611
*9.3 Singularity Functions to Determine Slope and Deflection 623
9.4 Method of Superposition 635
*9.5 Moment-Area Theorems 649
*9.6 Moment-Area Theorems Applied to Beams with Unsymmetric Loadings 664
Review and Summary 679

10 Columns 691

10.1 Stability of Structures 692
*10.2 Eccentric Loading and the Secant Formula 709
10.3 Centric Load Design 722
10.4 Eccentric Load Design 739
Review and Summary 750

11 Energy Methods 759

11.1 Strain Energy 760
11.2 Elastic Strain Energy 763
11.3 Strain Energy for a General State of Stress 770
11.4 Impact Loads 784
11.5 Single Loads 788
*11.6 Multiple Loads 802
*11.7 Castigliano's Theorem 804
*11.8 Deflections by Castigliano's Theorem 806
*11.9 Statically Indeterminate Structures 810
Review and Summary 823

Appendices A1

A Moments of Areas A2
B Typical Properties of Selected Materials Used in Engineering A13
C Properties of Rolled-Steel Shapes A17
D Beam Deflections and Slopes A29
E Fundamentals of Engineering Examination A30

Photo Credits C1
Index I1

Index

A

Allowable (working) load, 32–33
Allowable stress, 32–33
Allowable stress design, 723–728, 739, 751
Aluminum column design, 726
American Association of Safety and Highway Officials, 34
American Concrete Institute, 34
American Forest and Paper Association, 34, 726
American Institute of Steel Construction (AISC), 34, 723–724, 728
American standard beam (S-beam), 424
Angle of twist
  elastic range and, 151, 167–170
  circular shafts, 167–170, 224
  non-circular shafts, 210
  torque (τ) and, 151, 167–170
  tubes (thin-walled hollow shafts), 214
Anisotropic materials, 64, 134
Anticlastic curvature, 249, 335
Area (A)
  centroid of, A2–A7
  composite, A4–A7, A11–A12
  moments of, A2–A12
  radius of gyration, A8–A10
Average shearing stress, 11, 45, 422, 468
Average value of stress, 7, 44
Axial loading
  eccentric, 238–239, 291–295, 307–312, 337–338
  member stresses from, 7–10
  multiaxial, 95–97, 137
  oblique plane stresses from, 27–28
  plane of symmetry with, 291–295, 337
  pure bending and, 238–239, 291–295, 307–312, 337–338
  strain energy under, 764, 824
  stress and strain under, 55–145
  stress components under, 31
Axial stresses, 7–10
Axisymmetry of circular shafts, 151–152

B

Bauschinger effect, 66
Beam analysis and design, 344–415
  bending and, 344–415
  elastic section modulus (S) for, 347–348, 371–372, 396, 408, 410
  load and resistant factor design (LRFD), 373
  nonprismatic beams, 348, 396–401, 410
  prismatic beams, 371–376, 408
  relationships between load, shear, and bending moment, 260–368, 408
  shear and bending moment diagrams for, 348–354, 407–408
  shear and stress distributions, 347–348, 407
  sign convention for, 349
  singularity functions for shear and bending moment, 348, 383–391, 409
  step functions, 385, 409
  transverse loadings, 346–348, 407
Beam of constant strength, 396, 410
Beams. *See also* Beam analysis and design; Cantilever beams
  boundary conditions, 604, 679
  cantilever, 604–605
  deflection of, 598–689
  longitudinal shear on arbitrary elements, 437–439, 468
  normal stress in, 558–561, 591
  overhang, 604
  plastic deformations, 441–442, 469
  principal stresses in, 559–561, 591
  shearing stress in, 558–561, 591
  shearing stress distribution in, 347–348, 407, 422–431, 468
  simply supported, 604, 606
  singularity functions for, 348, 383–391, 409, 623–630, 681–682
  slopes and deflections of, A29
  span, 346
  statically determinate, 346–347, 407
  statically indeterminate, 347, 600, 611–617, 679–681
  thin-walled members, 439–466, 469
  unsymmetric loading of, 454–462, 469
Bearing stresses, connections with, 12, 45
Bearing surface, 12, 45
Bending
  beam analysis and design for, 344–415
  couple moment (M), 240–241
  modulus of rupture (R), 274
  prismatic members, 237–343
  pure, 237–343
  strain energy due to, 766, 824
Bending moment diagrams
  beam analysis using, 348–354, 407–408
  by parts for moment-area theorems, 654–659, 684
  sign convention for, 349
Bending moments
  couples, 240–241
  pure bending in symmetric members, 240–241, 334
  relationships with load and shear 260–368, 408
  singularity functions for, 348, 383–391, 409
Boundary conditions of beams, 604, 679
Breaking strength, 60–61
Bridges, design specifications for, 34
Brittle materials
  compression test for, 62
  concrete, 62, 134
  cracks, 511
  maximum-normal-stress criterion, 506
  Mohr's criterion for, 510–511, 549
  rupture of, 60–62
  stress and strain transformations, 509–511, 549
  stress-strain diagram determination of, 60–62, 133
  tensile test for, 60–61
  under plane stress, 509–511, 549
Buckling, 692–694
Bulk modulus (k), dilatation and, 97–99, 137

C

Cantilever beams
  deflection of, 604–605, 651–653, 684
  moment-area theorem for, 651–653, 684
  shearing stresses in, 426–427
Castigliano's theorem, 804–809, 826
Center of symmetry, 460
Centric load design, 722–732, 751
Centric loading, 9, 45
Centroid of the area, A2–A7
Circular shafts
  angle of twist, 151, 167–170, 224
  axisymmetry of, 151–152
  deformations in, 151–153
  modulus of rupture (R), 196, 225–226
  plastic deformation in, 195–204, 225–227
  residual stresses in, 199–204, 226–227
  shearing strain in, 153, 223
  stress concentrations in, 187–190, 225
  stresses in, 150–151, 153–161, 223–224
  torsion in, 148–204, 223–227
Coefficient of thermal expansion, 82, 136
Columns, 690–756
  allowable stress design, 723–728, 739, 751
  aluminum, 726
  buckling, 692–694
  centric load design, 722–732, 751
  critical load, 692–694, 750
  eccentric load design, 739–745, 751
  eccentric loading, 709–714, 751
  effective length, 698–700, 750
  Euler's formula for, 694–702, 750
  fixed ends, 698–700
  interaction method, 740–741, 751
  load and resistance factor design (LRFD), 728
  pin-ended, 694–697
  secant formula for, 711–712, 751
  slenderness ratio, 696, 750
  stability of structures and, 692–702
  structural steel, 723–724
  wood, 726
Combined loads, principal stresses under, 575–583, 592
Composite area (A)
  centroid of, A4–A7
  moment of inertia and, A11–A12

Composite materials, 259–262, 335
moduli of elasticity and, 259–260, 335
pure bending of members of, 259–262, 335
transformed section of, 260–335
Compression test, 62
Computation error detection, 16
Concentrated loads, 8, 346
Concrete
design specifications for, 34
stress-strain diagram for, 62, 134
Connections, bearing stress in, 12
Constant strength, 348
Coulomb's criterion, 509
Couple (bending) moments, 240–241
Cracks, 511
Critical load, 692–694, 750
Critical stress, 696
Curvature
analysis of curved members, 319–327, 338
anticlastic, 249, 335
pure bending and, 241–244, 319–327, 334–335, 338
radius of (ρ), 247, 334, 335
stresses and, 320–323, 338
transverse cross section, 248–252
transverse loading, 238–239, 334
Cylindrical pressure vessels, stresses in, 520–522, 549

D

Deflection of beams, 598–689, 790–795, 806–809
bending moment and, 600–601
boundary conditions and, 604, 679
Castigliano's theorem for, 806–809
deformation under transverse loading, 602–610, 679–680
elastic curve, equation of for, 603–606, 679
energy methods for, 790–795, 806–809
flexural rigidity (EI), 603–604, 679
method of superposition for, 601, 635–643
moment-area theorems for, 601–602, 649–659, 664–674, 682–684
singularity functions for, 623–630, 681–682
slope and, 607–608, 623–630, 681–682, A29
statically determinate, 635–636
statically indeterminate, 600, 611–617, 636–637, 679–681
work-energy method for, 790–795
Deformation (δ). *See also* Elastic deformation; Plastic deformation
axial loading and, 56–58, 68–69, 102–104, 119–122, 135, 139
bending in symmetric members, 241–244
circular shafts, 151–153, 195–204, 223, 225
deflection of beams under transverse loading, 602–610, 679–680
elastic behavior and, 68–69, 135
elastic range stresses and, 244–248, 334
multiaxial loadings, 95–96
per unit length, 57–58, 133
plastic behavior and, 65–67, 135
pure bending, 241–252
rectangular parallelepiped, 96
relationships of E, G, and ν, 102–104, 139
relative displacement for, 69
shafts, 148–149, 151–153, 209–218, 223, 225–227
strain energy and, 760–762, 823
transmission shafts, 148–149
torsion and, 148–149, 151–153, 209–218, 223, 225–227
Design considerations
allowable stress and, 32–33
allowable stress design, 723–728, 739, 751
centric load design, 722–732, 751
columns, 722–745, 751
eccentric load design, 739–745, 751
factor of safety, 32–34
impact loads, 786–787, 725–726
interaction method, 740–741, 751
load and resistance factor design (LRFD), 34, 728
power (P), 185–186, 225
specifications for, 34
stress (σ) and, 31–37
transmission shafts, 185–187, 225
ultimate strength, 31–32
working (allowable) load, 32–33
Dilatation (e), 98–99, 137
Dimensionless quantities, 58
Distributed loads, 346
Double shear, 12, 45
Ductile materials
breaking strength, 60–61
maximum-distortion-energy criterion, 508
maximum-shearing-stress criterion, 507–508
necking, 60
percent elongation, 61
percent reduction in area, 62
strain-hardening, 61
stress and strain transformations, 507–508, 548
stress-strain diagram determination of, 59–62, 133–134
ultimate strength, 60–61
yield, 59, 134
yield criteria, 507–508, 548
yield strength, 60–61, 134

E

Eccentric axial loading
analysis of, 307–312, 338
columns, 709–714, 751
forces of, 238–239
neutral axis, 292
plane of symmetry with, 291–295, 337
pure bending and, 238–239, 291–295, 307–312, 337–338
secant formula for, 711–712, 751
Eccentric load design, 739–745, 751
Eccentric loading, 8–9, 44. *See also* Eccentric axial loading
Effective length, 698–700, 750
Elastic behavior, 65–67, 134
plastic behavior compared to, 65–67
stress-strain diagrams for, 65–66, 134
Elastic curve
equation of, 603–606, 679
flexural rigidity (EI), 603–604, 679
load distribution and determination of, 609–610
Elastic deformation, 68–69, 135
Elastic flexural formulas, 245, 334
Elastic limit, 65, 134
Elastic range
angle of twist in, 167–170
internal torque and, 156
shearing stresses in, 153–160, 223–224
stresses and deformation, 244–248, 334
torsion and, 153–160, 167–170, 223–224
Elastic section modulus (S)
beam analysis and design for bending, 347–348, 371–372, 396, 408, 410
elastic range and cross section of members, 246, 335
nonprismatic beam design, 396, 410
prismatic beam design, 371–372, 408
Elastic strain energy, 763–769, 824
Elastic torsion formulas, 155–156, 223
Elasticity (E), modulus of, 63–65, 102–104, 134, 139
composite material members, 259–260, 335
Hooke's law and, 63–65, 134
pure bending and, 259–260, 335
relationships with G and ν, 102–104, 139
stress and strain directional relationships, 65, 134
Elastoplastic material
plastic deformation of, 119–122, 139
pure bending in members, 274–278, 338
torsion in circular shafts, 196–199, 226
Elementary work, 761
Endurance limit, 67–68, 135
Energy methods, 758–822
Castigliano's theorem, 804–809, 826
deflection by, 790–795, 806–809
elastic strain energy, 763–769, 824
impact loads, 784–787, 725–726
multiple loads and, 802–804
single-loaded members, 788–790, 826
statically indeterminate structures, 810–816
strain energy, 760–775, 823–725
strain-energy density, 762–763, 823
work and, 788–795, 802–804, 826
work-energy method, 790–795
Engineering materials, properties of, A13–A16
Engineering stress and engineering strain, 62
Equilibrium equations for problem solutions, 16, 46
Euler's formula, 694–702, 750
fixed-end columns, 698–700
pin-ended columns, 694–697

F

Factor of safety, 32–34, 46
Failure
brittle materials under plane stress, 509–511, 549
cracks, 511
design consideration of, 33
ductile materials, 507–508, 548
fracture criteria, 509–511, 549
stress and strain transformations, 507–513, 548–549
theories of, 507–513, 548–549
yield criteria, 507–508, 548

Fatigue, 67–68, 135
endurance limit, 67–68, 135
limit of, 67
repeated loadings and, 67–68
Fiber-reinforced composite materials, 64–65, 104–108, 134, 139
Hooke's law for, 64–65, 134
lamina, 64
laminate, 64–65, 105
matrix, 64
multiaxial loading, 105
stress-strain relationships for, 104–108, 139
Fillets, stress concentrations in, 117–118, 122
Fixed-end columns, 698–700
Flexural rigidity (EI), 603–604, 679
Force
internal, 10–12
shearing stresses and, 10–12
transverse, 44–45
Fracture criteria for brittle materials, 509–511
Free-body diagrams, 4–6, 16, 45
problem solutions from, 16, 45
two-force member stress analysis, 4–6
Fundamentals of Engineering Examination, A30

G

Gauge length, 58–59
General loading conditions, 28–31

H

Hertz (Hz), 185
Hollow shafts (tubes)
circular, 154–155, 158–160, 223–224
thin-walled non-circular, 211–216, 227
Homogeneous materials, 94
Hooke's law
axial loadings, 63–65, 134
fiber-reinforced composite materials and, 64–65
modulus of elasticity (E), 63–65, 134
modulus of rigidity (G), 100–101, 138
multiaxial loadings, 95–97, 137
proportional limit for stress, 64, 134
shearing stress and strain, 100–101, 138–139
Horsepower (hp), 185, 225

I

Impact loads, 725–726, 784–787
design for, 725–726, 786–787
energy from, 725, 784–785
In-plane shearing stress, 484, 547
Inertia, moments of, A8–A12
Interaction method, 740–741, 751
Internal forces, 10–12
Internal torque, 156
Isotropic materials, 64, 94, 134

L

Lamina, 64
Laminate, 64–65, 105
Lateral strain, 94–95, 137
Line of action, 8
Load and resistance factor design (LRFD), 34, 46, 373, 728
Load-deformation curve, 57
Loadings. *See also* Torsion
axial, 7–10, 27–28, 31, 55–145
beam deflection and, 651–653, 664–667, 684–685
bending and, 238–239, 346–348
centric, 9, 44
columns, 692–694, 709–714, 728, 750–751
combined, 575–583, 592
concentrated, 8, 346
critical, 692–694, 750
design considerations of, 32–34
distributed, 346
eccentric, 8–9, 44, 709–714, 751
factor of safety, 32–34, 46
general conditions, 28–31, 46
impact, 784–787, 725–726
line of action for, 8
moment-area theorems and, 651–653, 664–667, 684–685
multiaxial, 95–97, 137
plane stress and, 556–597
relationships with shear and bending moment, 260–368, 408
repeated, 67–68
reverse, 66–67
singularity functions for equivalent open-ended, 385–386, 410
singularity in, 383
stress and strain under, 55–145
stress components under, 29–31
stresses from, 7–10, 27–28, 44
symmetric, 651–653, 684
transverse, 238–239, 346–348
ultimate, 728
uniformly distributed, 346
unsymmetric, 454–462, 469, 664–674, 684–685
working (allowable), 32–33
Longitudinal shear on arbitrary beam elements, 437–439, 468

M

Macaulay brackets, 384–385, 387
Macroscopic cracks, 511
Matrix, 64
Maximum absolute strain, 244, 334
Maximum absolute stress, 244, 334
Maximum deflection, 666–667, 685
Maximum-distortion-energy criterion, 508
Maximum elastic moment, 275–276, 336
Maximum-normal-stress criterion, 506
Maximum-shearing-stress criterion, 507–508
Members
axial stress in, 7–10
bearing stress in, 12
shearing stress in, 10–12
two-force diagrams for, 4–6
stability of, 9
Membrane analogy, 210–211
Microscopic cracks, 511
Modulus
bulk (k), 97–99, 137
elastic section (S), 246, 335, 347–348, 371–372, 396, 408, 410
elasticity (E), 63–65, 102–104, 134, 259–260, 335
relationships of E, G, and ν, 102–104, 139
resilience (σ_y), 763, 824
rigidity (G), 100–101, 102–104, 138–139
rupture (R), 196, 225–226, 274
toughness (ϵ_R), 762–763, 823
Young's (E), 63
Mohr's circle
plane strain, 532–534, 550
plane stress, 492–502, 547
Mohr's criterion for brittle materials, 510–511, 549
Moment-area theorems
bending-moment diagrams by parts, 654–659, 684
cantilever beams, 651–653, 684
deflection and, 601–602, 649–659, 664–674, 682–685
first, 601, 649–650, 682
general principles of, 649–651, 664
maximum deflection and, 666–667, 685
second, 601, 650–651, 683
statically indeterminate beams, 668–674, 685
symmetric loadings and, 651–653, 684
unsymmetric loadings, 664–674, 684–685
Moment of inertia, A8–A12
Moments of areas, A2–A12
centroid of the area, A2–A7
composite area, A4–A7, A11–A12
first, A2–A10
moment of inertia of, A11–A12
moment of inertia, A8–A12
parallel-axis theorem, A10–A11
radius of gyration, A8–A10
second, A11–A12
Multiaxial loadings, 95–97, 105, 137
fiber-reinforced composite materials, 105
Hooke's law for, 95–97, 137
principle of superposition for, 96
rectangular parallelepiped deformation from, 96
Multiple loads, work and energy under, 802–804

N

Neutral axis, 243, 334
Neutral surface, 242–243, 334
Non-circular shafts, 209–216, 227
angle of twist, 210
membrane analogy for, 210–211
thin-walled (tubes), 211–216, 227
torsion in, 209–216, 227
uniform rectangular cross sections, 210, 227
Non-rectangular cross sections, plastic deformation in, 277
Nonprismatic beams
analysis and design for bending, 348, 396–401, 410
elastic section modulus (S) for, 396, 410
Normal strain, axial loading and, 57–58, 133

Normal stress
beams, 558–561, 591
determination of, 7, 44
maximum criterion for brittle materials, 506
strain energy and, 763–767, 824
Numerical accuracy, 16

O

Oblique parallelepiped deformation, 99–100
Oblique planes, stresses on from axial loading, 27–28, 46
Orthotropic materials, 105
Overhanging beam, 604

P

Parabolic beam, 424–425
Parallel-axis theorem, A10–A11
Parallelepipeds
oblique, 99–100
rectangular, 96
Percent elongation, 61
Percent reduction in area, 62
Permanent set, 65, 134. *See also* Plastic deformation
Pin-ended columns, 694–697
Plane strain, 529–537, 549–550
Mohr's circle for, 532–534, 550
three-dimensional analysis of, 534–537
transformation equations, 529–531, 549
transformation of, 529–534, 546–550
Plane stress, 478–479, 480–506, 546
Mohr's circle for, 492–502, 547
principal stresses, 482–487, 546
state of, 478–479
three-dimensional analysis, 504–506
transformation equations for, 480–482, 546
transformation of, 480–487, 546
Plastic behavior, 65–67, 134
elastic behavior compared to, 65–67
permanent set, 65, 134
stress-strain diagrams for, 65–66, 134
reverse loadings and, 66–67
Plastic deformation
Bauschinger effect, 66
beams, 441–440, 469
circular shafts, 195–204, 225–227
creep, 65
elastic behavior and, 65–67, 134
elastic limit, 65–66, 134
elastoplastic material, 119–122, 139
elastoplastic members, 274–278, 336
members with single plane of symmetry, 278–279
modulus of rupture (R), 196, 225–226, 274
non-rectangular cross sections, 277
permanent deformation and, 200–201, 226–227
permanent set, 65–67, 134
pure bending and, 273–285, 336
rectangular cross sections, 274–277
residual stresses, 122–126, 139, 199–204, 226–227, 279, 336
reverse loadings and, 66–67
single-plane symmetric members, 278–279
slip, 65
stress and strain under axial loads, 65–67, 119–122, 134, 139
stress concentrations and, 122
thin-walled members, 440–441, 469
torsion and, 195–204, 225–227
Plastic hinge, 441
Plastic moment, 275–276, 336
Polar moment of inertia, A8
Poisson's ratio (ν), 94–95, 102–104, 137
lateral strain and, 94–95, 137
relationships with E and G, 102–104, 139
Power (P) transmitted by shafts, 185–186, 225
Pressure vessels, 520–524, 549
cylindrical, 520–522, 549
spherical, 522, 549
stresses in, 520–524, 549
thin-walled, 520–524, 549
Principal planes of stress, 482–487, 546
Principal strains, 523
Principal stresses, 483, 546, 556–597
beams, 559–561, 591
combined loads and, 575–583, 592
loading and, 556–597
plane stress transformation and, 483, 546
transmission shaft design for, 562–569, 592
Prismatic beams, design for bending, 371–376, 408
Prismatic members, pure bending of, 237–343
Problems
computation error detection, 16
equilibrium equations for, 16, 46
free-body diagrams for, 16, 45
numerical accuracy of, 16
Saint-Venant's principle for, 115–117, 139
SMART methodology for, 15–16
solution, method of, 15–19, 45–46
statically equivalent, 115–117
statically indeterminate, 78–81, 135–136
superposition method for, 79–81
temperature changes and, 82–88, 136–137
Properties of materials, A13–A28
Proportional limit for stress, 64, 134
Pure bending, 237–343
composite members, 259–262, 335
curved members, 319–327, 338
deformations from, 241–252
eccentric axial loading, 238–239, 291–295, 307–312, 337–338
elastic range stresses and deformation, 244–248, 334
elastoplastic members, 274–278, 336
members with single plane of symmetry, 278–279
plastic deformations, 273–285, 336
prismatic members, 237–343
residual stresses from, 279–280
stress concentrations from, 263–267, 336
symmetric members in, 240–244, 334
transverse cross sections, 248–252
transverse loading, 238–239, 334
unsymmetric bending analysis, 302–307, 337

R

Radius of curvature (ρ), 247, 334
Radius of gyration, A8–A10
Rectangular cross sections, plastic deformation in, 274–277
Rectangular parallelepiped deformation, 96
Redundant reactions, 810
Relative displacement, 69,
Repeated loadings, fatigue from, 67–68
Residual stresses
circular shafts, 199–204, 226–227
permanent deformation and, 200–201, 226–227
plastic deformation and, 122–126, 136, 199–204, 226–227, 279
pure bending and, 279, 336
temperature change and, 124
torsion and, 199–204, 226–227
Resilience (u_r), modulus of, 763, 824
Resistance factor (ϕ), 728
Reverse loadings, plastic behavior and, 66–67
Rigidity (G), modulus of, 100–101, 102–104, 138–139
Hooke's law and, 100–101, 138
relationships with E and ν, 102–104, 139
shearing strain and, 100–101, 138
Rolled steel shapes, properties of, A17–A28
Rotation, speed of, 185
Rupture (R), modulus of, 196, 225–226, 274
Rupture of brittle materials, 60–62

S

Saint Venant's criterion, 509
Saint Venant's principle, 115–117, 139
Secant formula, 711–712, 751
Section modulus (S), *see* Elastic section modulus
Shafts, 147–234
circular, 148–204, 223–227
deformation of, 148–149, 151–153, 209–218, 223, 225–227
hollow (tubes), 154–155, 158–160, 223–224
non-circular, 209–216, 227
plastic deformation of, 195–204, 225–227
residual stresses in, 199–204, 226–227
statically indeterminate, 170–176, 225
stresses in, 150–151, 153–161, 209–218, 223–224
thin-walled hollow (tubes), 211–216, 227
torsion in, 147–234
transmission, 148–149, 185–187, 225
Shear
double, 12, 45
relationships with loads and bending moments, 260–368, 408
single, 11, 45
ultimate strength in, 32
Shear center, 419, 455, 469
Shear diagrams, 348–354, 407–408
beam analysis for bending, 348–354, 407–408
sign convention for, 349
Shear flow, 419, 421, 440–441, 468
Shear moments, singularity functions for, 348, 383–391, 409
Shearing strain
axial loading and, 99–102, 138

circular shafts, 153, 223
Hooke's law for, 100–101, 138
modulus of rigidity (G), 100–101, 138
oblique parallelepiped deformation, 99–100
Shearing stresses
average, 11, 45, 422, 468
beam design for, 417–475
beams, distribution of in, 347–348, 407, 422–431, 468
bending and, 347–348, 407
circular shafts, 153–160, 223
components of, 30–31
elastic range with, 153–160
forces exerted on transverse prismatic beams, 418–419, 467
horizontal, 419–426, 467
in-plane, 484, 547
internal force and, 10–12
longitudinal, 437–439, 468
maximum criterion for ductile materials, 507–508
plastic deformation and, 441–446
points of application, 44
strain energy due to, 767–769, 824
thin-walled member design for, 439–466, 469
unsymmetric loading and, 454–462, 469
vertical, 418, 467
Simple structures, analysis and design of, 12–15
Simply supported beam, 604, 606
Single-loaded members, work of, 788–790, 826
Single shear, 11, 45
Singularity functions
application to computer programming, 388
beams, 348, 383–391, 409, 623–630, 681–682
bending analysis and design using, 348, 383–391, 409
equivalent open-ended loadings for, 385–386, 410
Macaulay brackets, 384–385, 387
shear and bending moments using, 348, 383–391, 409
slope and deflection using, 623–630, 681–682
Singularity in beam loading, 383
Slenderness ratio, 696, 698, 750
Slope and deflection
beams, 607–608, 623–630, 681–682, A29
relationship of, 607–608
singularity functions for, 623–630, 681–682
SMART methodology, 15–16
Span, 346
Speed of rotation, 185
Spherical pressure vessels, stresses in, 522, 549
Stability
critical load, 692–694, 750
members, 9
structures, 692–702
Stable system, 692–693
Statically determinate members
beams, 346–347, 407, 635–636, 682
deflection and, 635–636, 682
method of superposition and, 635–636, 682
Statically equivalent problems, 115–117
Saint-Venant's principle, 115–117
uniform distribution of, 116–117
Statically indeterminate members
beams, 347, 600–601, 611–617, 636–637, 668–674, 679–682
deflection and, 611–617, 636–637, 668–674, 697, 680–682
first degree, 381, 612
forces, 56
method of superposition and, 636–637, 682
moment-area theorems for, 668–674, 685
problems, 78–81, 135–136
second degree, 381, 612
shafts, 170–176, 225
stress distribution, 8, 56
Statically indeterminate structures, 810–816
Statics
free-body diagrams, 4–6
review of methods, 4–6
Steel, design specifications for, 34
Step functions, 385, 409
Strain (ε). *See also* Stress and strain transformations; Stress and strain under axial loading
bending, 244, 334
circular shafts, 153, 223
engineering, 63
lateral, 94–95, 137
maximum absolute, 244, 334
measurement of, 538–541
normal, 57–58, 133
plane, 529–537, 549–550
Poisson's ratio (ν), 94–95, 102–104, 137
shearing, 99–102, 138, 153, 223
thermal, 82, 136
true, 63
Strain energy, 760–775, 823–725
axial loading and, 764, 824
bending and, 766, 824
deformation and, 760–762, 823
elastic, 763–769, 824
general state of stress and, 770–775, 825
modulus of resilience and, 763, 824
modulus of toughness and, 762–763, 823
normal stresses and, 763–767, 824
shearing stresses and, 767–769, 824
torsion and, 767–768, 825
transverse loading and, 769
Strain-energy density, 762–763, 823
Strain-hardening, 61
Strain gages, 480, 538, 547
Strain rosette, 480, 538, 547
Stress. *See also* Stress and strain transformations; Stress and strain under axial loading
allowable, 32–33
applications to analysis and design of simple structures, 12–15
average value of, 7, 44
axial, 7–10, 31
beams, distribution of in, 347–348, 407
bearing, 12, 45
bending, 244–248, 334, 347–348, 407
circular shafts, 150–151, 153–160, 223–224
components of, 28–31
concept of, 2–53
critical, 696
curved members, 320–323, 338
defined, 7
direction of the component, 29
design considerations, 31–37
elastic range deformation and, 244–248, 334
engineering, 63
exerted on a surface, 29
factor of safety, 32–34, 46
general loading conditions, 28–31, 46
internal forces and, 10–12
load and resistance factor design (LRFD), 34, 46
loadings and, 7–10, 27–31
maximum absolute, 244, 334
method of problem solution, 15–19, 45–46
normal, 7, 44
oblique planes under axial loading, 27–28, 46
plane, 478–479, 480–506, 546
proportional limit, 64, 134
residual, 122–126, 139
shearing, 10–12, 30–31, 44–45, 153–160
statically indeterminate distribution of, 8
true, 63
ultimate strength, 31–32
uniaxial, 242
uniform, 44
Stress and strain transformations, 476–555
brittle materials, 509–511, 549
ductile materials, 507–508, 548
failure, theories of, 507–513, 548–549
general state of stress, 503–504, 548
in-plane shearing stress, 484, 547
measurement of strain, 538–541
Mohr's circle for, 492–502, 547
plane strain, 529–537, 549–550
plane stress, 478–479, 480–506, 546
states of stress, 478–479
thin-walled pressure vessels, 520–524, 549
three-dimensional stress analysis, 504–506
yield criteria, 479
Stress and strain under axial loading, 55–145
bulk modulus (k), 97–99, 137
deformations from, 56–57, 68–69, 102–104, 119–122, 135, 139
dilatation, 97–99, 137
elastic limit, 65, 134
elastic versus plastic behavior, 65–67
endurance limit, 67–68, 135
fatigue from, 67–68, 135
fiber-reinforced composite materials, 64–65, 104–108
Hooke's law, 63–65, 95–97, 100–101, 134
lateral strain, 94–95, 137
modulus of elasticity (E), 63–65, 102–104, 134
modulus of rigidity (G), 100–101, 102–104, 138–139
multiaxial loadings, 95–97, 137
normal strain, 57–58, 133
plastic deformation, 65–67, 119–126, 134, 139
Poisson's ratio (ν), 94–95, 102–104, 137
repeated loadings, 67–68
residual stresses, 122–126, 139
Saint-Venant's principle, 115–117, 139
shearing strain, 99–102, 138
statically equivalent problems, 115–117
statically indeterminate problems, 78–81, 135–136
stress concentrations, 117–118, 122, 139
stress-strain diagram, 58–62, 65–67, 133–134
temperature change effects on, 82–88, 136–137
true stress and true strain, 63
uniform distribution of, 116–117

Stress concentrations
circular shafts, 187–190, 225
circular stress distribution, 117–118
discontinuity factor (K), 117–118, 139
fillets, 117–118, 122
flat stress distribution, 117–118
plastic deformations and, 122
pure bending, 263–267, 336
torsion and, 187–190, 225
Stress-strain diagrams, 57–62, 65–67, 133–134
axial loading and, 57–62, 65–67, 133–134
breaking strength, 60–61
brittle material determination, 60–61, 133
compression test for, 62
ductile material determination, 59–62, 133–134
gage length of specimen, 58–59
load-deformation curve, 57
rupture and, 60–61
tensile test for, 58–62
ultimate strength, 60–61
yield strength, 60–61, 134
Structural steel column design, 723–724
Superposition
method of for deflection, 601, 635–643, 682
multiaxial problems, 96–97
principle of, 96
statically determinate beams, 635–636, 682
statically indeterminate beams, 636–637, 682
statically indeterminate problems, 79–81
Symmetric loadings, moment-area theorems for, 651–653, 684
Symmetric members, 240–244, 334
bending (couple) moments in, 240–241, 334
deformation from pure bending, 241–244, 334
plastic deformation in, 278–279

T

Temperature change
coefficient of thermal expansion, 82, 136
plastic deformation and, 124
problems involving, 82–88, 136–137
residual stresses and, 124
stress and strain under axial loads and, 82–88, 124, 136–137
thermal strain, 82, 136
Tensile test, 58–62
Thermal expansion, coefficient of, 82, 136
Thermal strain, 82, 136
Thin-walled members
beam design for shearing stresses, 439–466, 469
non-circular (tubes), 211–216, 227
plastic deformations in, 440–441, 469
shear flow, 440–441
shearing stresses in, 439–466, 469
unsymmetric loading of, 454–462, 469
Three-dimensional analysis
strain, 534–537
stress, 504–506
Timber, design specifications for, 34
Torque (T), 148, 151, 156, 167–170
Torsion, 147–234
angle of twist, 151, 167–170, 210, 214, 224
circular shafts, 148–204, 223–227
elastic range, 153–160, 167–170, 223–224
elastoplastic materials, 196–199, 226
hollow shafts (tubes), 154–155, 158–160, 223–224
modulus of rupture (R), 196, 225–226
non-circular shafts, 209–216, 227
plastic deformation and, 195–204, 225–227
residual stresses from, 199–204, 226–227
shearing stresses from, 153–160, 167–170, 209–218, 224–227
strain energy due to, 767–768, 825
stress concentrations and, 187–190, 225
stresses in, 150–151, 153–161, 223–224
thin-walled hollow shafts (tubes), 211–216, 227
transmission shafts, 148–149, 185–187, 225
Torsion formulas
elastic range, 155–156, 223
variable circular cross sections, 188, 225
Toughness (e_R), modulus of, 762–763, 823
Transformed section, 240
Transmission shafts
deformation of, 148–149
design of, 185–187, 225, 562–569, 592
power transmitted by, 185–186, 225
principle stresses and, 562–569, 592
speed of rotation of, 185
Transverse cross sections, 248–252
Transverse forces, 44–45
Transverse loading
beam analysis and design for, 346–348, 407
concentrated, 346
deflection of beams under, 602–610, 679–680
distributed, 346
pure bending and, 238–239, 334
shear and stress distributions, 347–348, 407
strain energy under, 769
support reactions, 347–348
uniformly distributed, 346
True stress and true strain, 63
Tubes, 211–216. *See also* Shafts
Two-force member analysis, 4–6

U

Ultimate load, 728
Ultimate strength, 31–32, 60–61
Uniaxial stress, 242
Uniform distribution of stress and strain, 116–117
Uniform stress, 44
Uniformly distributed loads, 346
Unstable system, 692–693
Unsymmetric bending analysis, 302–307, 337
Unsymmetric loading
beam deflection and, 664–674, 684–685
maximum deflection and, 666–667, 685
moment-area theorems for, 664–674, 684–685
thin-walled members, 454–462, 469

V

Volume change, 97–99, 137
axial loadings and, 97–99, 137
bulk modulus (k) for, 97–99, 137
dilatation, 98–99, 137
von Mises criterion, 508

W

Watts (W), 185, 225
Wide-flange beam (W-beam), 424
Wood column design, 726
Work, 788–795, 802–804, 826
deflection by, 790–795, 806–809
energy and, 788–795, 802–804, 826
multiple loads and, 802–804
single-loaded members, 788–790, 826
Work-energy method, 790–795

Y

Yield, 59, 134
Yield criteria for ductile materials, 507–508, 548
Yield points, 61
Yield strength, 60–61, 134
Yielding, design consideration of, 33
Young's modulus (E), 63. *See also* Elasticity