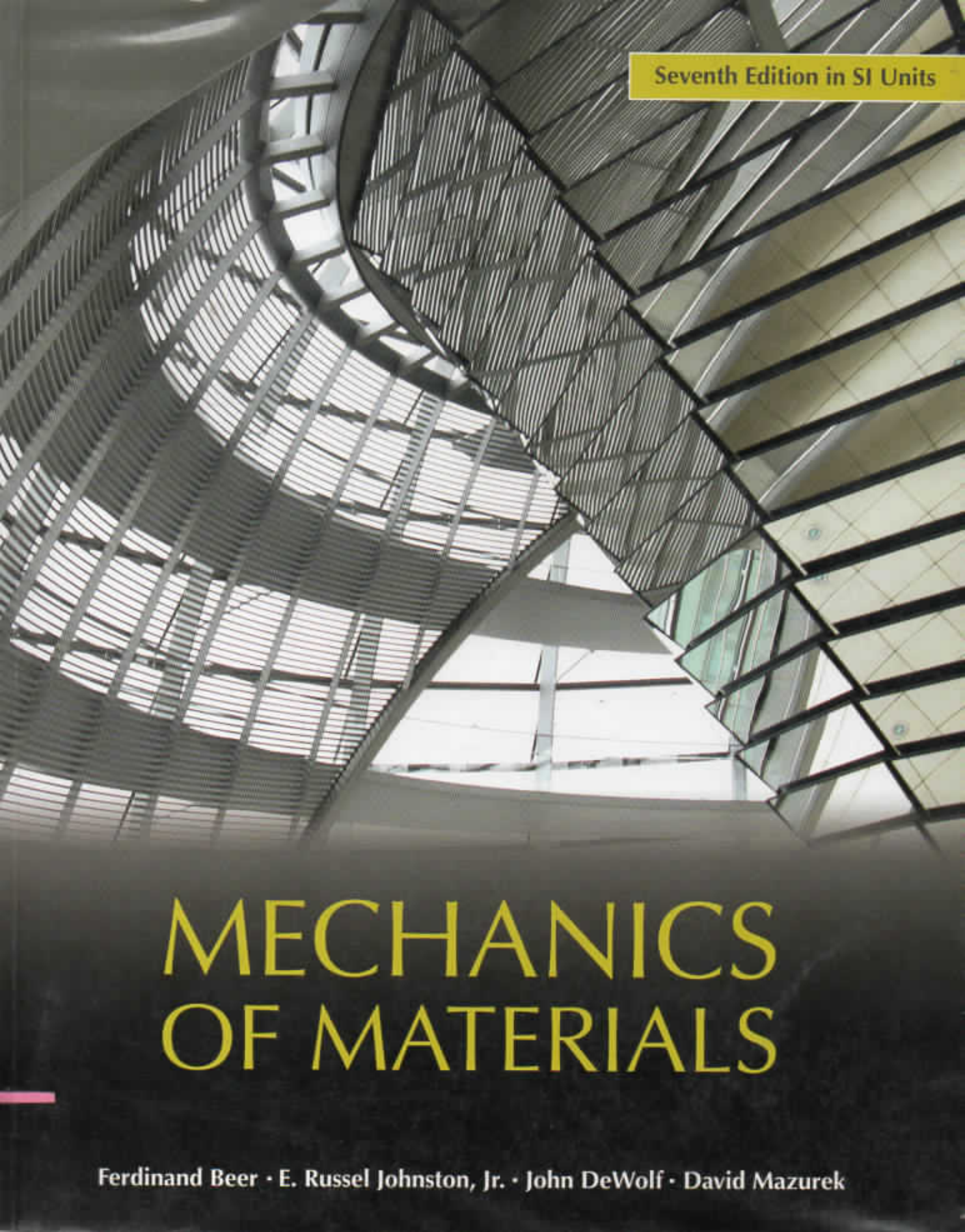


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MECHANICS OF MATERIALS

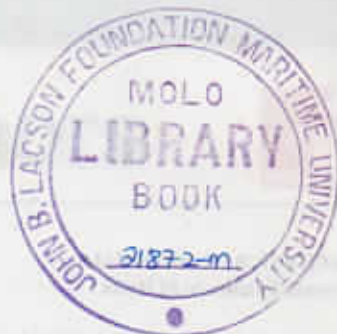
Ferdinand Beer • E. Russel Johnston, Jr. • John DeWolf • David Mazurek



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Mechanics of Materials

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